EPR effect in gravitational field:
nature of non-locality[*]

Horst von Borzeszkowski[a†]
Michael B. Mensky[ab‡]

[a] Institut für Theoretische Physik, Technische Universität Berlin
Hardenbergstr. 36, D-10623 Berlin, Germany

[b] P.N.Lebedev Physical Institute, 53 Leninskii prosp,
117924 Moscow, Russia

February 4, 2000

Abstract

The realization of the Einstein-Podolsky-Rosen effect by the correlation of spin projections of two particles created in the decay of a single scalar particle is considered for particles propagating in gravitational field. The absence of a global definition of spatial directions makes it unclear whether the correlation may exist in this case and, if yes, what directions in distant regions must be correlated. It is shown that in a gravitational field an approximate correlation may exist and the correlated directions are connected with each other by the parallel transport along the world lines of the particles. The reason for this is that the actual origin of the quantum non-locality is founded in local processes.

1 Introduction

The Einstein-Podolsky-Rosen (EPR) effect (paradox) [1] demonstrates, as is commonly believed, a sort of quantum non-locality because the measurement of one of the two spatially separated systems changes the state of the other provided that they are in an entangled state. This can be described as a quantum correlation of the systems. We shall clarify the nature of this non-locality by considering the EPR effect for spinning particles propagating in an external gravitational field. It is not evident that the correlation must exist in this case because there is no 'natural' correspondence between the directions in the two spatially separated regions in a curved space-time. We shall however see that an approximate correlation may exist even in this case and that the correlated directions are connected with each other by the parallel transport along the world lines of the particles.[1] This proves that the apparent non-locality is in fact of a local nature.

It is well known that the EPR effect may be realized with the help of two spin 1/2 particles forming in the decay of a scalar particle and therefore having correlated spin projections. In this case the measurement of the spin projection of the first particle resulting say in $m_z^{(1)} = 1/2$ (spin up) makes the spin projection of the other particle definite, $m_z^{(2)} = -1/2$ (spin down), which may be verified by its measurement.

This phenomenon is well understood and easily explained by the consideration of the state of both particles after the decay:

$$\frac{1}{\sqrt{2}} \left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right). \tag{1}$$

This so called entangled state means that the projections of the spins of both particles on the same axis z are correlated: the measurement of them always gives opposite results. After some period of free evolution the particles may have moved far away from each other, but the spin state of both particle and therefore the correlation of the spin projections will be the same.

The localization of the particles in two spatially separated regions (the key point for the EPR paradox) is no obstacle for a correct formulation of such a correlation 'in ordinary conditions'. Indeed, the statement "two particles

---

[1]In quantum mechanics the concept of a world line may be introduced in the framework of an approximation, see later.

located in different space regions have opposite spin projections on axis z" has quite a definite meaning under the condition that the gravitational field acting on the particles is negligible.

The situation however radically changes if the particles move in a substantial gravitational field. In this case the statement "If $m_z^{(1)} = 1/2$, then $m_z^{(2)} = -1/2$" has no sense for spatially separated particles, because, in the absence of teleparallelism, the axis z in the region where the second particle is located may be chosen independently of the z axis in the vicinity of the first particle.

The following questions naturally arise: 1) Does a quantum correlation maintain between the spins of two particles when the particles move in a gravitational field and 2) if it exists, how may this correlation be specified (what directions in the two distant regions are correlated)? We shall consider these questions first on the intuitive level and then in the framework of the path-integral description of a spinning particle in an external gravitational field (in a curved space-time). The conclusion will be that the correlation is in this case approximate and has to be formulated with the help of the parallel transport along the trajectories (world lines) of the correlated particles.

2   Arguments of plausibility

Let in a space-time point O a spin-0 particle decay forming two particles of spin $1/2$, which then propagate to the space-time points A_1, A_2. If this takes place in the flat (Minkowski) space-time (see Fig. 1 left), then, because of the teleparallelism, a single, globally defined reference frame may be defined in the space-time. The axis z chosen in point A_1 may correctly be used in an arbitrary point in the whole space-time. Therefore, no question arises as to what is the z-direction in the point A_2. The correlation between the two particles may be described in terms of this common z-direction as it has been made in Introduction.

If however the space-time is curved (Fig. 1 right), then the z-direction defined in a certain manner in point A_1 does not determine any z-direction in other points, among them in point A_2.

The reason for this difficulty is that there is no global notion of the z-direction or more generally of spacial directions or space-time directions. Such notions may be introduced in an arbitrary (space-time) point A with
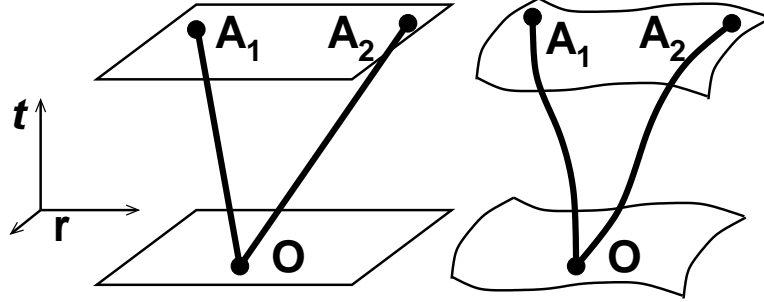
Figure 1: Two spinning particles propagating in the flat (left) or curved (right) space-time. In the latter case there is no globally defined notion of z-direction. Instead one has to use local frames. However the natural (in respect to the given particles) correspondence between the local frames in the points A_1 and A_2 may be established by the parallel transport along the curve A_1OA_2 consisting of the world lines of the particles.

the help of a local frame n, which is a basis of the tangent space to the curved space-time in the given point. However local frames in different points may be chosen independently of each other, so there is no natural correspondence between the local frame n_1 in the point A_1 and the local frame n_2 in the point A_2.

The correlation between the spin projections in the points A_1 and A_2 can be formulated only if some unambiguous correspondence is established between local frames n_1 and n_2 in the points A_1 and A_2 in such a way that the choice of one of them determines the other. This correspondence must be based on some essential features of the physical process under consideration.

If a certain curve connecting the points A_1 and A_2 is chosen, then the correspondence between n_1 and n_2 may be established with the help of the operation of parallel transport along this curve. It is quite evident that the only relevant curve for this aim is the curve A_1OA_2 composed of the world lines of the particles, so it is natural to choose this curve for the parallel transport of the local frames.

Thus, it seems plausible that the EPR correlation in a gravitational field is the following. Choose an arbitrary local frame n_1 in point A_1 and define the spin projection of the first particle with respect to the z-axis of this local frame. Then perform the parallel transport of this local frame to the

point O along the curve A_1O (the world line of the first particle but passed backwardly in time) and then along the world line OA_2 of the second particle to the point A_2. A local frame n_2 in the point A_2 results. Define a spin projection of the second particle with respect to the z-axis of this frame. The correlation formulated in Introduction should exist between the thus defined spin projections.

This scheme seems plausible, but of course it must be proved in the framework of the dynamical consideration. Particularly, it has to be justified 1) that the operation of parallel transport must be taken to identify z-directions in the locations of the two particles and 2) that the curve for this parallel transport must be composed of the world lines of the particles. Moreover, in the plausible formulation we used the purely classical notion of a world line of a particle which may only approximately be applied in the case of quantum particles. These points will be made more precise in the next section in the course of the dynamical consideration on the basis of the path- integral approach. It will be shown that the above-mentioned correlation may actually arise for particles in a gravitational field, but only approximately, if the particles are localized around the points O, A_1, A_2 in regions which are narrow in comparison with the distances OA_1 and OA_2. In the general case the correlation of spin projections emerging in the decay is destroyed during the propagation of the particles in the gravitational field.

3   Dynamical proof

In this section we shall shortly introduce a formalism for the description of a relativistic spinning particle moving in a curved space-time. Then this formalism will be applied to analyze EPR corralations in a curved space-time.

3.1   The evolution of a relativistic spinning particle

Let $\psi(x)$ (with x being a space-time point) is an arbitrary solution of the (covariant) Klein-Gordon (KG) equation and $U(x, x')$ an arbitrary Green function of this equation:

$$(\Box + m^2)\psi(x) = 0, \tag{2}$$
$$(\Box + m^2)\,U(x, x') = -i\delta(x, x'). \tag{3}$$

Then for an arbitrary four-dimensional region Ω having a characteristic function $\eta_\Omega(x)$ (the function which is equal to unity in Ω and zero outside) the following relation holds:

$$- i \int_{\partial\Omega} \sigma^{\mu'} U(x, x') \overleftrightarrow{\nabla}_{\mu'} \psi(x') = \eta_\Omega(x)\psi(x). \tag{4}$$

Here the three-dimensional integration over the boundary $\partial\Omega$ of Ω is performed with the measure determined by the three-form

$$\sigma_\mu = \frac{1}{6}\sqrt{-g(x)}\, \epsilon_{\mu\nu\sigma\lambda}\, dx^\nu \wedge dx^\sigma \wedge dx^\lambda, \tag{5}$$

∇_μ is a covariant derivative and $\overleftrightarrow{\nabla}_\mu$ denotes

$$\varphi(x) \overleftrightarrow{\nabla}_\mu \psi(x) = \varphi(x)\nabla_\mu\psi(x) - (\nabla_\mu\varphi(x))\psi(x).$$

The relation (4) may be derived from Eq. (3).

Let Ω be a 4-dimensional region containing the point x and restricted by two space-like surfaces, one in the future and the other in the past of x. Then the integral (4) is performed over these two surfaces. However for a positive-frequency solution of the KG equation (describing a particle but having no counterpart related to an antiparticle)[2] and for $U(x, x')$ being the causal propagator the integral over the future surface is zero. Therefore, only the integral over the past surface S survives in this case (this may be interpreted as the property of positive frequencies to propagate only forward in time):

$$\psi^{(+)}(x) = i \int_{S<x} \sigma^{\mu'} U(x, x') \overleftrightarrow{\nabla}_{\mu'} \psi^{(+)}(x'). \tag{6}$$

This is the evolution law for a relativistic particle which may have nonzero spin. To analyze the problem of correlation we need to investigate properties of the propagator $U(x, x')$. We shall do this with the help of the path-integral presentation of this propagator.

---

[2]The positive-frequency function is defined in the region of interest (where the particle propagates) if a time-like Killing vector field exists in this region.

3.2 The path integral for the propagator

Different and not always equivalent definitions of the path integral in a curved space-time may be given [2, 3, 4]. We shall use the definition of [3] which is equivalent to that one in [4].

It turns out that it is much simpler to give a path-integral representation not for the causal propagator $U(x, x')$ (which is a 2-point function) but for the corresponding integral operator U defined as follows:

$$(U\varphi)(x) = \int d^4x \sqrt{-g(x)}\, U(x, x')\, \varphi(x'). \tag{7}$$

The path-integral definition of this operator may be given in the following way [3]. First the operator U may be expressed in terms of the operator U_τ depending on 'proper time' τ (which is sometimes called historical time or simply fifth parameter [5]):

$$U = \int_0^\infty d\tau\, e^{-im^2\tau} U_\tau. \tag{8}$$

The operator U_τ may be expressed in the form of a path integral:

$$U_\tau = \int d[\xi]\, e^{(-i/4) \int_0^\tau d\tau\, \dot{\xi}^\alpha \dot{\xi}_\alpha}\, D[\xi]. \tag{9}$$

Here the integration is carried out over all paths $[\xi]$ in Minkowski space. Paths may be defined as elements of the group of paths (classes of continuous curves) [6] or simply as continuous curves having the fixed initial point (say, the origin of Minkowski space) and an arbitrary final point. The covariant displacement operator $D[\xi]$ is defined as an ordered exponential of the integral along $[\xi]$:

$$D[\xi] = P \exp\left(\int_{[\xi]} d\xi^\alpha\, \nabla_\alpha \right). \tag{10}$$

This is in fact the operator of parallel transport along those curves in the curved space-time which have the curve $[\xi]$ (in Minkowski space) as their evolvent, see [3, 7]. The integral in Eq. (10) includes the operator of the covariant derivative but carrying the Lorentzian index α instead of the world index μ :

$$(\nabla_\alpha \varphi)(x) = n_\alpha^\mu(x)\, (\nabla_\mu \varphi)(x). \tag{11}$$

7

This formula makes use of a field $n(x) = \{n_\alpha^\mu(x)\}$ of local frames (a section of the fiber bundle of orthonormal local frames) which may be chosen arbitrarily and then used in this and the subsequent formulas. For the spinning particle corresponding to the representation D of the Lorentz group, the covariant derivative ∇_μ with the world index μ is defined as follows[3]:

$$(\nabla_\mu \psi)(x) = \left[\frac{\partial}{\partial x^\mu} + \widetilde{D}(M_\mu(x))\right] \psi(x) \tag{12}$$

where $\widetilde{D}$ is the representation of the Lorentzian Lee algebra corresponding to the representation D of the Lorentz group. The matrix $M(x)$ belonging to the Lorentzian Lee algebra is

$$[M_\mu(x)]_\beta^\gamma = \left(n^{-1}\right)_\nu^\gamma \left[\frac{\partial n_\beta^\nu(x)}{\partial x^\mu} + n_\beta^\sigma \Gamma_{\mu\sigma}^\nu(x)\right].$$

The 'propagator depending on the proper time' $U_\tau(x, x')$ satisfies the relativistic Schrödinger equation:

$$(\Box - i\frac{\partial}{\partial \tau}) U_\tau(x, x') = 0. \tag{13}$$

It has no direct physical interpretation, but it is exploited as an intermediate step for the definition of the causal propagator $U(x, x')$. As a consequence of Eq. (13) and the definition (8), the causal propagator $U(x, x')$ satisfies Eq. (3), i.e., it is a Green function of the KG equation.

3.3   Analysis of the propagator

We see that the evolution (6) of the particle is described by the propagator which is expressed in the form of a path integral by Eqs. (7, 8, 9) containing the operator (10) of parallel transport. Let us assume that the initial state of the particle is localized in a rather small region of the spacelike surface S which may be, in a reasonable approximation, presented by a single point (the point O in Fig. 1) and the final state is also concentrated in a small region (near the point A_1 or A_2) of the corresponding space-like surface.

---

[3]We make use here of the conventional definition of the covariant derivative instead of the equivalent definition in terms of the fiber bundle of local frames which has been used in [3]

It is known that the main contribution to the path integral is given by those paths which are close to geodesic lines (this is valid if the initial and final points are not too close to each other). Let us assume (this assumption will be discussed later) that the uncertainties of the position in the initial and in the final state of each particle are small in comparison with the distance between its initial and final positions. Then all the paths which substantially contribute to the path integral for the given particle are close to the same geodesic, namely the geodesic line connecting the point of the initial localization O of the particle with the point of its final localization (A_1 or A_2). Let us denote the corresponding geodesics as $\gamma_1 = OA_1$ and $\gamma_2 = OA_2$. Therefore, all paths essentially contributing to the evolution of the particle are close to γ_1 for the first particle and close to γ_2 for the second particle.

The contribution of each path in Eq. (9) is expressed through the parallel transport along this path. For the paths giving the main contribution (close to the geodesic γ_1 or γ_2) the corresponding parallel transports are close to the parallel transport along the corresponding geodesic, γ_1 for the first particle and γ_2 for the second particle. Therefore, we are led to the following conclusion: If the initial spin state of the particle is described by a function referred to some local frame n_0 in the initial point O, then the final spin state is approximately described by the same function but referred to the local frame (in the final point A_1 or A_2) which is obtained from n_0 by the parallel transport along the corresponding geodesic γ_1 or γ_2. Let us denote the resulting local frames n_1 for the first particle and n_2 for the second particle.

Summing up, the spin state of the first particle in the final point A_1 is described by the same function as in the initial point O but if the initial spin state refers to the local frame n_0 and the final spin state refers to the local frame n_1 obtained from n_0 by parallel transport along γ_1. The same is valid also for the second particle, but with n_2 and γ_2 instead of n_1 and γ_1.[4]

This evidently justifies the statement made in Sect. 2 about EPR correlations in a gravitational field. Indeed, the correlation between spin states

---

[4]This statement may be technically elaborated, most conveniently with the help of wave functions defined as functions on the fiber bundle of local frames. A spin state of a localized particle may be described by a function on the corresponding fiber of local frames in the point of localization. If a reference local frame is given, then the function on the fiber (representing the spin state) is reduced to the function on the Lorentz group. The same function on the Lorentz group but referring to the parallelly transported local frame will present the final spin state of the particle.

of the two particles is formed at the moment of their formation at a single point O. Then the particles propagate to different points A_1, A_2. If this propagation may, in a good approximation, be presented as the propagation of the localized states along the geodesics γ_1 and γ_2, then the spin state of each particle will be parallelly transported into the new point of localization along the corresponding geodesic. The correlation of the initial spin states is described with reference to the same local frame n_0 (at the point O which is common for both particles). Therefore the correlation between the final spin states must be described in just the same way but with reference to the parallelly translated local frames (correspondingly n_1 and n_2). The relation between these local frames is evident: n_2 may be obtained from n_1 by the parallel transport backward along the world line $OA_1 = \gamma_1$ of the first particle (which results in n_0) and then forward along the world line $OA_2 = \gamma_2$ of the second particle. Finally n_2 is the parallel transport of n_1 along the curve $A_1OA_2 = (\gamma_1)^{-1}\gamma_2$.

The preceding conclusions were based on the assumption that the initial state of each particle is localized in a rather small region and the propagation of each particle may, in a good appoximation, be presented by a single path coinciding with the corresponding geodesic line. Let us discuss this assumption.

At the first glance it is hardly possible to justify the assumption that the evolution of each particle may be accurately enough presented by only one geodesic γ_1 or γ_2. Indeed, because of the uncertainty of the initial linear momentum the localization in the final point is poorer than it is in the initial point; the longer period of propagation the poorer the localization. The particle may be considered localized about the final point A_1 or A_2 only if the period of propagation is rather short. However for a short period of evolution it is impossible to neglect the paths which strongly deviate from geodesics. This leads to a contradiction which hardly may be resolved.

In this latter argument we refer to the final localization which is the result of the free evolution. This localization is poor because of the spreading of the wave packet. However, what we really need is the localization of the particles in the measurement here under consideration. Let not only the spin projections, but simultaneously with this the coordinates of the particles are measured. Then the resulting uncertainty of the coordinate will be determined by the precision of the measurement of the coordinate. In other words, in the process of the measurement of the spin projection the

particle may be localized in an arbitrarily narrow region about the point A_1 or A_2 (these points are determined only in the process of this localization during the measurement of the spin projections). In this case only those paths in the path integral are relevant which lead to this region. We may conclude then that only a rather narrow bunch of paths contributes to that component of the state which is essential for the analysis. If the region of the final localization is narrow enough, then all relevant paths are close to the corresponding geodesic (γ_1 or γ_2), so the assumption we have accepted in the analysis of the spin corelation is justified.

This completes the dynamical proof of the statement which has been fromulated in Sect. 2 on the basis of plausible arguments.

It is of course possible that one meets another situation when the assumption taken above is invalid. Let the localization of the particles (initially or finally or both) be not enough strong to justify the approximation of a single path. Then we have to apply a more strict approximation taking into account more than one path of propagation of each particle. In this case the initial correlation between the spin projections of the two particles (necessarily resulting in the decay of a scalar particle) is destroyed in the course of the propagation. The longer the propagation and the stronger the gravitational field, the poorer is the correlation. In general the EPR correlation between spin projections does not exist.

4 Conclusion

The conclusion drawn from the previous consideration may be formulated as follows. What is usually called quantum non-locality (or the correlation of the EPR type) is in fact a correlation between the results of the measurements of two spatially separated systems. However this apparently non-local phenomenon is preceded by 1) establishing the correlation between the two systems in a single point O (locally) and then 2) bringing the correlated systems to the separated points A_1 and A_2 during the evolution (propagation). Both operations are local. Thus, the final correlation is prepared in the course of the local processes.

This is made explicit by the above consideration of the EPR effect in an external gravitational field. We have seen that the spin states of the particles in the points A_1 and A_2 are correlated if they are referred to such local frames

n_1 and n_2 in these points that n_2 results from the parallel transport of n_1 along the line A_1OA_2 consisting of the two geodesics, OA_1 and OA_2.

Thus, in the case of the gravitational field (when there is no teleparallelism) the very specification of the correlation may be made only in terms of the world lines (classical trajectories) of the particles (essentially local objects). In general no EPR correlation exists in a gravitational field. The reason for this is that different local processes (essentially different paths) are coherently superposed.

References

[1] A. Einstein and B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935).

[2] B.S.DeWitt, Reviews of Modern Physics, **29**, 377 (1957).

[3] M.B.Mensky, Theor. Math. Phys. **18**, 136 (1974).

[4] V.Pažma and P.Prešnaider, Czechosl. J. Phys. B38, 968, (1988); H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, and Polymer Physics*, World Scientific, Singapore, 1995.

[5] E.C.Stückelberg, Helv. Phys. Acta **14**, 322 (1941); **15**, 23 (1942).

[6] M.B.Mensky, *The Path Group: Measurements, Fields, Particles*, Nauka, Moscow, 1983 (in Russian; Japanese translation: Yoshioka, Kyoto, 1988); M.B.Mensky, Applications of the path group in gauge theory, gravitation and string theory, in: *Gauge Theories of Fundamental Interactions* , World Scientific, Singapore etc., 1990, p.395.

[7] M.B.Mensky, The equivalence principle and symmetry of Riemannian space, in: *Gravitation: Problems and Prospects*, the memorial volume dedicated to A.Z.Petrov, Kiev, Naukova Dumka publishers, 1972, p.157 (in Russian); M.B.Mensky, Helvetica Physica Acta 69, 301-304 (1996)); Quantum Equivalence Principle, in: "Group-21", H.-D.Doebner et al. (Eds.), World Scientific, Singapore etc., 1997, vol.I, p.331.